On numerical modeling of subsidence induced by mining

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ABSTRACT: The paper contains a new approach to calculate subsidence induced by mining. Its essence consists in combining reliable empirical data with rigorous accounting for compatibility conditions in 3D multi-layered media. The mathematical tool employed is the fast Fourier transform. It serves us for efficient solution of 3D elasticity equations for tens and even hundreds of layers. Stresses at the Earth surface are zero. At the mining levels, displacement discontinuities are considered to be proportional to the extracted thickness. Low- or no-friction contacts are presumed. Normal displacements at contacts may be continuous or include contact dilation. The layers are considered to be isotropic, homogeneous and elastic. Asymptotic formulae for high and low frequencies are used to increase accuracy and decrease the time of calculations. Layers being normally thin, the accepted presumptions do not distort the subsidence picture significantly given the boundary conditions at the mining level are prescribed in terms of displacements. This allows us to introduce necessary corrections accounting for strongly nonlinear effects discussed in the paper. The corrections are made automatically in the computer program Lay-Subsid developed by the authors. Numerical results and examples are given. They show flexibility and potential of the suggested approach for subsidence prediction and for further development of the code.

INTRODUCTION

Subsidence of the Earth surface induced by mining is a vital environmental problem in many countries including France and Russia. Extensive researches have been carried out on this subject to the date (see, e. g. recent reviews in [1,2]). They embrace vast empirical data (e.g. [3-5]), semi-empirical approaches (e.g. [3,5-7]), mathematical interpretations and numerical tools (e.g. [3,6-10]). Purely empirical approaches, being reliable for simple geometry, fail to provide extensions to a great variety of practical situations. Their inclusion into analytical and/or numerical frame is desired. Models employing influence functions (e.g. [3,7]) and the laminated model by Salamon [8,9] suggest such inclusion. The latter model, containing usual influence functions as a particular case, gives even values within an entire stratum. However, this model also has limitations concerning with layer thickness, mining at various levels and accounting for contact dilation, dip angle, faults, etc. There is need to develop new tools, which retaining virtues of the mentioned models, may account for the additional important factors.

In this paper, we suggest such an extension. It consists in solving numerically 3D problems for multi-layered media with displacement discontinuities on contacts and, in perspective, across layers. It employs recent advance in numerical solution of problems for layered systems [11-15]. In Sec. 1 and 2 we present essential features of its implementation for subsidence problems. A specially developed methodology to account for strongly nonlinear effects is described. Physical meaning of correction parameters is explained.

The program Lay-Subsid, developed by the authors, employs this approach. Results and examples obtained by using this program are given in Sec. 3. Conclusion contains brief summary of the results. In whole, the paper reflects the first stage in developing the suggested approach. We show that, being more flexible than existing methods, it successfully covers areas available to them.

1. MATHEMATICAL FORMULATION OF THE PROBLEM AND A METHOD OF SOLUTION

Bending of layers into an extracted space, being an obvious feature of subsidence process, it looks reasonable to use a model of a layered stratum. First, we suppose layers to be *elastic*. Necessary corrections will be made later. We consider 3D layered system with horizontal boundaries of layers. In problems of geomechanics the upper boundary of the package is free of traction. Its lower boundary is in contact with an elastic half-space. On contacts of layers we have (i) equilibrium conditions for traction and (ii) prescribed dependence between vectors of traction and displacement discontinuities (these vectors are connected by a matrix of contact interaction).

Some of contacts correspond to mining levels. We prescribe contours of mined out areas on them. Within these contours, the roof and the floor are free of traction, if they are not in contact. Otherwise, a law of interaction may be prescribed to account for particular interaction. Emphasize, however, that in subsidence problems, we are interested in *displacements* rather than in stresses. That is why we are trying to satisfy *compatibility* equations first of all. Consequently, in these problems we choose to prescribe *displacement discontinuities* at the mining level. They are taken equal to the extracted thickness multiplied by a coefficient which accounts for nonlinear processes in a roof.

For a pillar between mined out areas, a law of its deformation may be also prescribed. In many subsidence problems, one may assume the displacement discontinuities to be zero along a pillar.

This data completely defines mathematical formulation of a problem. To solve it, we employ the Fourier transform combined with the highly efficient method of reduction to the three-point difference equations solved by the sweep method [12]. The efficiency additionally increases when employing the fast Fourier transform. Besides, we use asymptotic formulae specially derived for high and low Fourier frequencies. They allow us to increase accuracy and to decrease time spending. The code Lay-Subsid, developed by the authors, incorporates all these effective means. It allows us to solve 3D problems for tens and even hundreds of layers with tens of mined out areas. As a result, we obtain a tool to tackle the subsidence problem in a way mentioned in Introduction.

2. CORRECTIONS SERVING TO ACCOUNT FOR NON-LINEAR EFFECTS

The main obstacle to apply rigorous methods to subsidence prediction is that the subsidence is controlled by strongly nonlinear processes. The most obvious of them are (a) roof failure, (b) contact sliding, (c) crack growth and opening in tensile zones of layers, (d) movements along faults. Consider these factors in their sequence.

(a) The *roof failure* causes uncertainties in the effective thickness of an extracted layer. Consequently, this thickness should be taken in accordance with empirical data for a particular mining region. We account for it in a way used in mining industry codes: by introducing three multipliers to a real thickness to obtain an effective thickness. The latter is used as boundary condition. The first of the multipliers reflects the influence of mining technology, namely, whether and what kind of filling is used. Actually, it presents the ratio of the observed subsidence to the extracted thickness for *super-critical* areas of extraction[1]. The second coefficient gives the Proust correction for a depth when it exceeds 400 m [2,5,10]. The third multiplier, normally taken equal to unit, enables corrections for local peculiarities when empirical data on subsidence is available.

(b) *Contact sliding* leaves uncertain the thickness and the number of layers which experience mutual movements on their contacts. We account for this factor by imposing no-friction conditions on the contacts of layers. The number of such layers above a mined out area is chosen so, that the *width* of subsidence profiles for super-critical areas agrees

[1] The terms “sub-critical”, “critical” and “super-critical” for extracted areas have the meaning common in publications on subsidence (see, e. g. [4]). The critical area normally corresponds to W/H = 1.4. For the sub-critical area W/H <1.4; for the super-critical area W/H > 1.4.

with empirical data. Surprisingly enough, this number, being in our estimation about fourteen, is rather stable for coal seams in various countries.

In additional to the width of a subsidence profile, we adjust the position of its inflection point. Empirical data for super-critical areas show that this point is not exactly above the seam edge. According to the observations [4,6], it is at a distance d of about $(0.1\text{-}0.15)H$ from the edge towards an opening. This shift reflects the fact that the roof is not in contact with the floor immediately at the edge of a seam. To account for this factor, one may use a smaller contour at the distance d from the edge of a seam. Naturally, $2d$ cannot exceed the width W of an extracted area. Consequently, for small values of the width to depth ratio ($W/H < 0.6$), the distance d is taken depending on this ratio. We suggest two options for choosing this dependence. One of them employs data of the British Coal Board [4], the other that attributed to Proust [2,5,10]. This distinction is necessary because of significant difference between the maximum subsidence observed in British and French mining regions for the same small value $W/H = 0.2$ [2].

These adjustments result in profiles which are in good *quantitative* agreement with the generalized empirical data embraced by mining codes. They also give the maximum subsidence for *sub-critical* areas in agreement with measurements for long-wall mining. This provides us with an important prerequisite for extending calculations beyond the simple schemes covered by empirical data.

The other important feature, facilitating extensions, is that the numerical method employed preserves ***additivity***. It means that the subsidence induced by a number of extracted elements is equal to the sum of their individual inputs. The additivity, being observed in field measurements, gives us a good chance to have reliable results for complicated contours composed of simple parts for which we have obtained agreement with empirical data.

(c) *Opening of cracks* in tensile zones of layers strongly influences the measured *horizontal* displacements and strains at the Earth surface. Their influence may be explained as following. Suppose we have a solid, without cracks, kernel layer that defines a profile, and slender ribs attached to its surface that experiences tension. The ribs imitate behavior of a cracked zone. The thickness of the solid kernel is $2h_{\text{ker}}$, the length of the ribs is h_{rib}. For bending, when a cross section rotates remaining plane, the measured tangential strain is proportional to the distance from the neutral axis of the kernel. Hence, the strain measured along the upper boundary of the ribs is $k_{\text{strain}} = (h_{\text{ker}} + h_{\text{rib}})/h_{\text{ker}}$ fold greater than the strain on the surface of the solid kernel layer. Consequently, we need to multiply horizontal deformations resulting from the data on profiles, by a correction coefficient k_{strain}.

Analysis of empirical data presented in [4,6] shows that the multiplier k_{strain} varies from 2.2 to 4.5 depending on particular mining region. Its usual value is about 3. This coefficient is prescribed in our code as an input parameter to have horizontal displacements and strains in agreement with the observations for simple geometry of an extracted area.

Note that the influence of soft surface soil, having low both tensile and compressive strength, is similar to that of cracks but it includes also zones of compression. The empirical coefficient k_{strain} reflects this influence as well. Note also that this coefficient may be taken different for tensile and compressive horizontal strains.

(d) Influence of *faults* may be accounted for by employing the techniques of the spectral BEM. The necessary mathematical prerequisites are given in [11, 13, 14]. As the techniques allow us to include boundaries not parallel to layer surfaces, it may serve to account for the influence of the dip angle as well. We hope to account for this factor in this way in future work.

Summarizing the said, four corrections are made to agree our numerical results with the reliable empirical data for super-critical areas. Specifically,

(i) the displacement discontinuities at an extraction level are prescribed in a way which provides the maximum value of subsidence in agreement with the observations;

(ii) the number of layers is taken in a way providing the width of a subsidence profile in agreement with the observations;

(iii) the contour of a mined area is corrected to provide the observed position of the inflection point and the measured maximum subsidence for sub-critical areas.

These three corrections concern with the vertical displacements and their derivatives (slopes and curvatures). The forth correction, as explained, concerns with the horizontal displacements and their derivatives (horizontal strains):

(iv) the horizontal strains and displacements obtained for a calculated profile are multiplied by k_{strain} to account for cracks and soft soil in a way providing agreement with the measured horizontal strains.

The corrections are made automatically in the computer program Lay-Subsid developed by the authors.

3. RESULTS AND EXAMPLES

Figures 1-4 illustrate relation between calculated results (solid lines) and empirical data (dashed lines). The first of them (Fig. 1) presents maximum subsidence as a function of width to depth ratio.

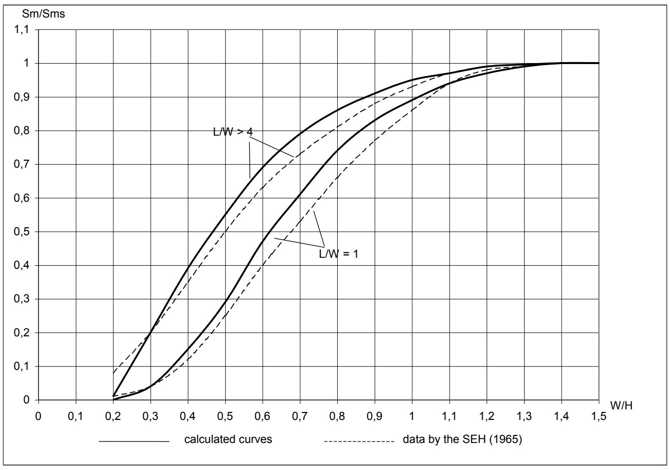

Fig. 1. Dependence of the maximum subsidence on W/H

It is compared with the SEH prediction [4]. The maximum subsidence S_m is normalized by its limit value S_{ms} corresponding to a super-critical area of extraction (W/H > 1.4). For the upper pair of the curves, the length L of a mined area is great enough not to influence the results (L/W > 4). In this case, as explained in Sec. 3, the position of the inflection point is chosen to meet the empirical data. No wonder that the agreement is satisfactory. More remarkable is that the results for square mined areas, shown by the lower pair of the curves, are also in reasonable agreement. Note that analogous agreement is also reached for a so called Proust prediction [2,10].

Fig. 2 presents profiles of subsidence for sub-critical (L/W = 0.6), critical (L/W =1.4) and super-critical (L/W =2.0) areas. The empirical data is that of the handbook [4]. Again we may see reasonably good agreement between calculated and empirical profiles. Note, that, as could be expected, for the supercritical area, the profiles simply translate along the abscissa approximately reproducing those for the critical area.

The influence of more complicated geometry is illustrated by Fig. 3, 4. Fig. 3 shows an extracted area (thick line) in the mine Estaque Sud of the Provence coal district of France. The depth is 1000 m; the thickness of a mined seam is 2.85 m. Solid circles mark points along the observation path where subsidence was measured. Thin lines give the calculated isolines of subsidence for this case. In general, they follow contours of the mined area. Fig. 4 presents the measured and calculated subsidence along the path of observations. The agreement looks satisfactory.

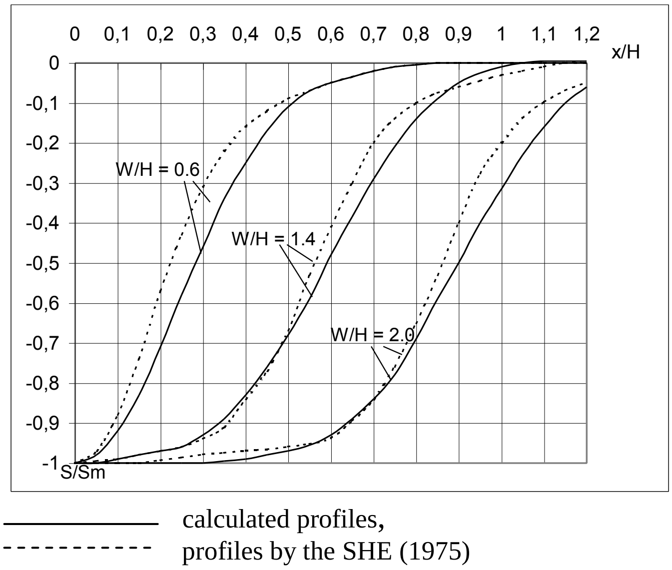

Fig. 2. Profiles of subsidence for sub-critical (L/W=0.6), critical (L/W=1.4) and super-critical (L/W=2.0) areas

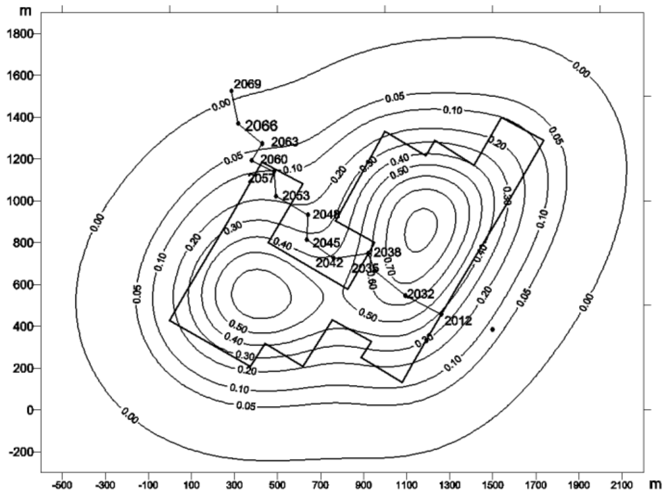

Fig. 3. Calculated isolines of subsidence for the mine Estaque Sud in the Provence coal district of France

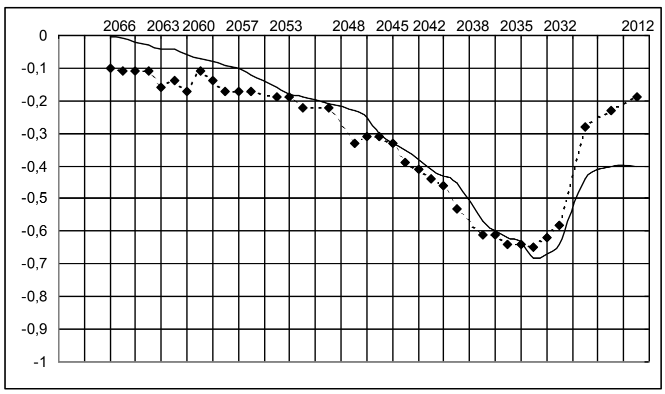

Fig. 4. Measured (diamonds) and calculated (solid line) subsidence along the path of observations in the mine Estaque Sud in the Provence coal district of France

4. CONCLUSION

We conclude that the method developed and the program Lay-Subsid may serve for subsidence prediction. It easily reproduces empirical data for simple geometry and it preserves additivity. This allows us to use it for complicated geometry of extracted areas.

The arguments of Sec. 2, analysis of empirical data and gained experience convince us that whatever sophisticated method to calculate subsidence may give reliable results only when employing *four empirical corrections* of the types mentioned above. As a matter of fact, any method successfully used in practice employed them explicitly. If such a method keeps additivity, it suggests a tool to predict subsidence for complicated geometry of mined areas. In this sense, the classical method by Knothe [5] and that by Salamon [8,9] are very close to our approach. The difference is in a degree of flexibility. The approach developed has greater flexibility in accounting for layered structure, contact conditions, faults and dip angle. Providing a convenient tool for subsidence prediction, it suggests also additional means for further development.

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